Quantum *versus* Classical Turbulence

Julien Salort

Thesis work carried out at

Institut Néel, CNRS, Grenoble

Hydro team : *Philippe-E. Roche, Benoît Chabaud, Bernard Hébral, Christoph Weise, Mathieu Gibert, Hichem Benalia*

Supervisor :

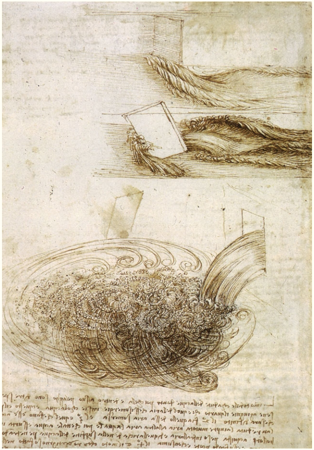
Philippe-E. Roche

Collaborations :

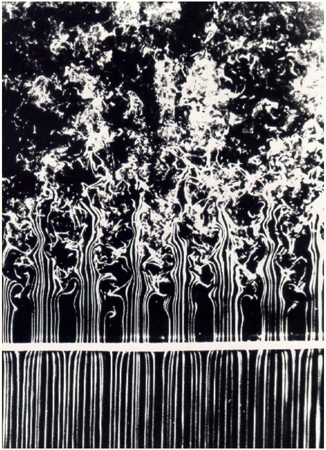
Alessandro Monfardini, Emmanuel Lévêque, Collaboration TSF

Turbulence...

ie. the dynamics of a “strongly” stirred fluid...

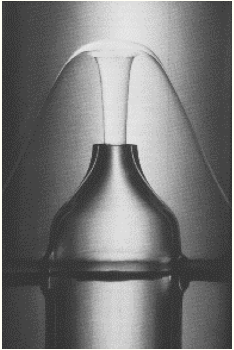


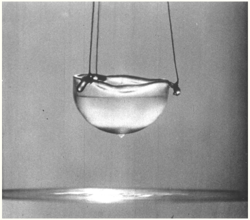
Leonard de Vinci, xvith century

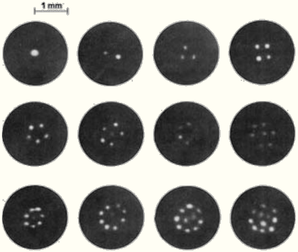


H.M. Nagib, Fluid Dynamics Research Center

…superfluid



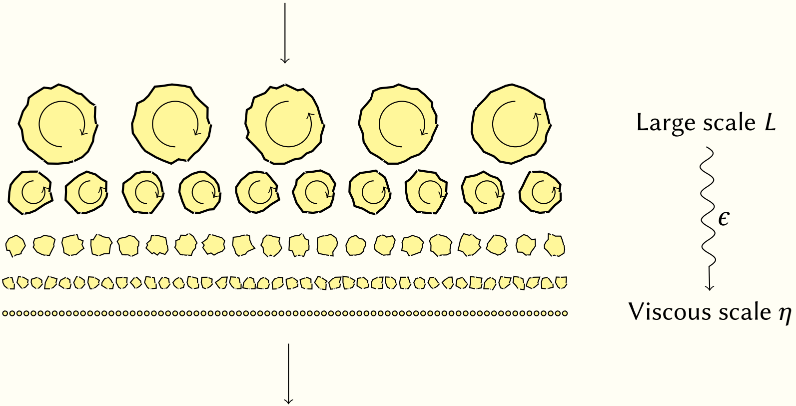




Exotic fluid :

- regarded as invisid ;
- quantized circulation of velocity.

Classical turbulence : the inertial cascade



Energy injected at large scale

Large scale L

ϵ

Viscous scale η

Energy dissipated at smale scale

Control parameter

Reynolds number

- Defined from the large scale L

$$Re = \frac{L v_{\mathrm{rms}}}{\nu}$$

- Defined from Taylor micro-scale λ :

$$R_\lambda = \frac{\lambda v_{\mathrm{rms}}}{\nu}$$

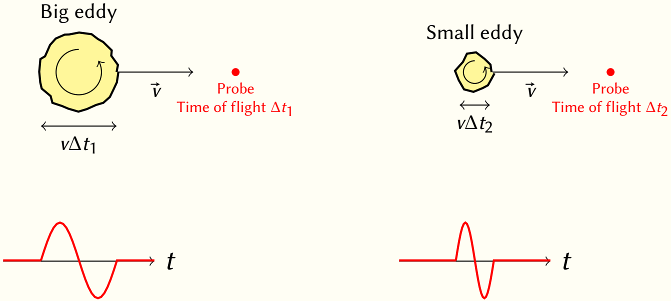
with

$$\langle \|\nabla v\|^2 \rangle = \left(\frac{v_{\mathrm{rms}}}{\lambda} \right)^2$$

Turbulence

$$\boxed{Re \gg 1}$$

Informations on eddies : Taylor hypothesis

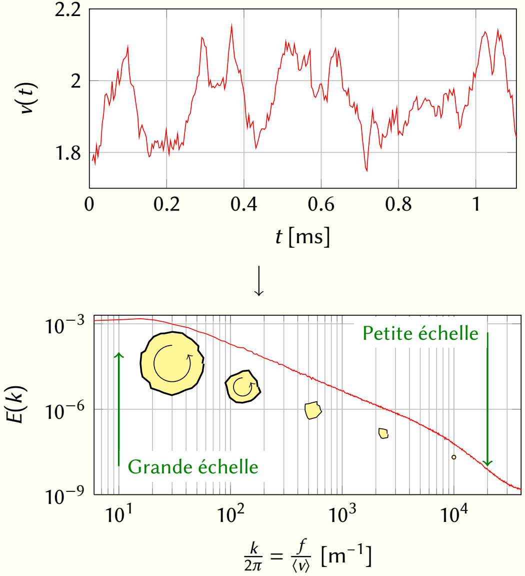


- A frequency f corresponds to a eddy of size v/f
- Wave number : $k = \frac{2\pi f}{v}$

Consequence on velocity spectrum

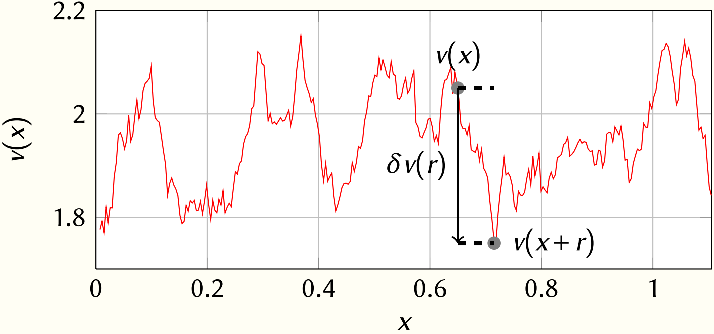
Measured signal

Energy distribution among eddies

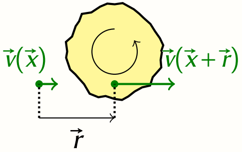


Longitudinal velocity increments statistics

$$\boxed{\delta v(r;x) = v(x+r) - v(x)}$$



1941 Kolmogorov theory : -5/3 spectrum



Typical velocity :

$$\delta v = v(x+r) - v(x)$$

Instable structure :

Lifetime ≈ Turnover time

$$\epsilon = \frac{\text{Energy per unit mass}}{\text{Turnover time}} \approx \frac{\langle \delta v^2 \rangle}{r/\sqrt{\langle \delta v^2 \rangle}} \approx \frac{\langle \delta v^2 \rangle^{3/2}}{r}$$

$$\langle \delta v^2 \rangle \sim \epsilon^{2/3} r^{2/3}$$

Formulation in Fourier space

Kolmogorov's law :

$$E(k) = C_k \epsilon^{2/3} k^{-5/3}$$

1941 Kolmogorov theory : energy cascade

Energy flux across the scales

An analytical derivation from the Navier-Stokes equation leads to

$$\boxed{\langle \delta v^3 \rangle = -\frac{4}{5}\epsilon r + 6\nu \frac{\partial \langle \delta v^2 \rangle}{\partial r}}$$

This formula is often cited as the only exact relation in turbulence. It leads to a non-symmetric distribution for δv.

Interpretation as an energy budget :

$$\epsilon = \underbrace{-\frac{5}{4}\frac{\langle \delta v^3 \rangle}{r}}_{\text{Transfer (cascade)}} \underbrace{-\frac{15\nu}{2r}\frac{\partial \langle \delta v^2 \rangle}{\partial r}}_{\text{Dissipation}}$$

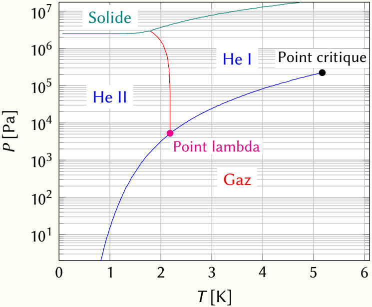
Cryogenic helium as a working fluid

$$Re = \frac{L v_{\mathrm{rms}}}{\nu}$$

- Liquid helium as a classical viscous fluid

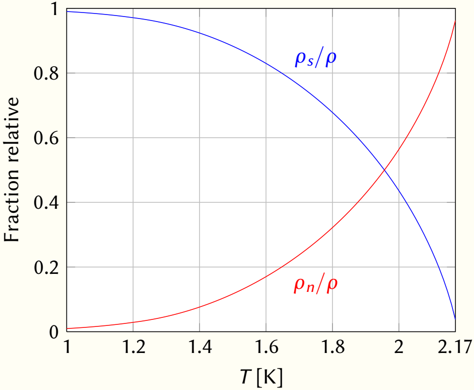
Fluide	T [K]	P [bar]	$\nu = \eta/\rho$ [m^2/s]
Air	293	1	1.5×10^{-5}
Eau	293	1	1.0×10^{-6}
SF_6	300	15	1.5×10^{-7}
$He_{(g)}$	4.2	1	7.4×10^{-8}
$He_{(l)}$	4.2	1	2.6×10^{-8}

- Phase transition at $T_\lambda \approx 2.17\,\mathrm{K}$

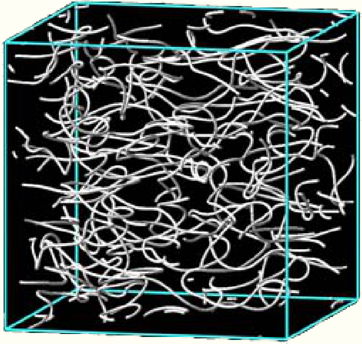


He II : two-components fluid

- Normal component (n) : viscous
- Superfluid component (s) :
  - inviscid
  - quantized velocity circulation (κ)
  - irrotationnal except along quantum vortex lines



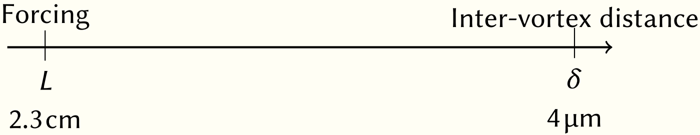
System of interest : superfluid ^{4}He at large Re



Adachi et al. 2010

- How is energy dissipated ?
- Is there a hierarchy of scales ?
- Is there a Kolmogorov cascade ?
- How analogous is it to a classical flow ?

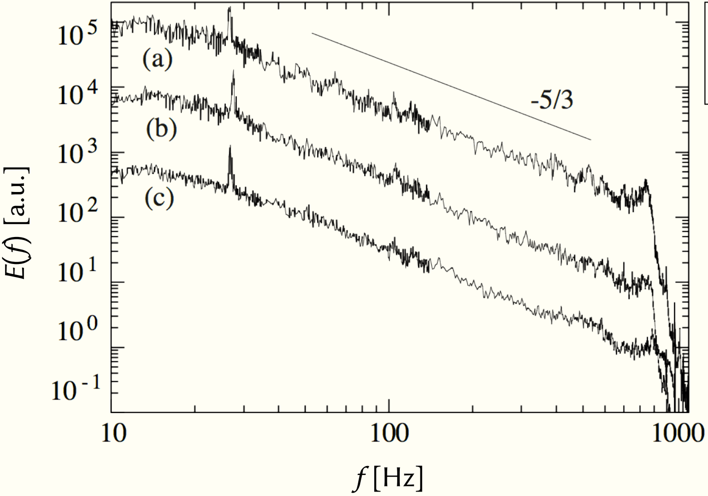
Typical scales ($R_\lambda \approx 1000$)



Velocity fluctuations : superfluid cascade ?

Local velocity fluctuations (steady turbulence)

Maurer & Tabeling, 1998



(a) 2.3 K
(b) 2.08 K ($\rho_s/\rho_n = 0.4$)
(c) 1.4 K ($\rho_s/\rho_n = 12$)

Main thread of this work :

Problem :

What are the similarities and the differences between classical turbulence and the turbulence in superfluid ^{4}He ?

Experimental challenge :

- Design of dedicated probes :
  - Low temperature specifications
  - Local velocity fluctuations measurement : our aim is to reduce the probe size from 1 mm to 100 µm
- Design of dedicated wind tunnels :
  - High Reynolds number
$R_\lambda \approx 300$ for the grid flow
$R_\lambda \approx 1000$ in a pipe
  - $1.2\,\mathrm{K} < T < 2.17\,\mathrm{K}$
  - Good hydrodynamical quality

Works carried out during this thesis

Probe developments :

- Miniature Pitot tubes ;
- Cantilever probe ;
- Second-sound tweezers.

Wind tunnels :

- Test wind tunnel ;
- Counterflow wind-tunnel ;
- TSF facilities ;
- Toupie wind tunnel.

Physical results :

- Velocity spectrum at inertial scales ;
- Energy cascade across the scales ;
- Velocity spectrum at small scales (numerical simulations) ;
- Temperature dependance of the vorticity spectrum.

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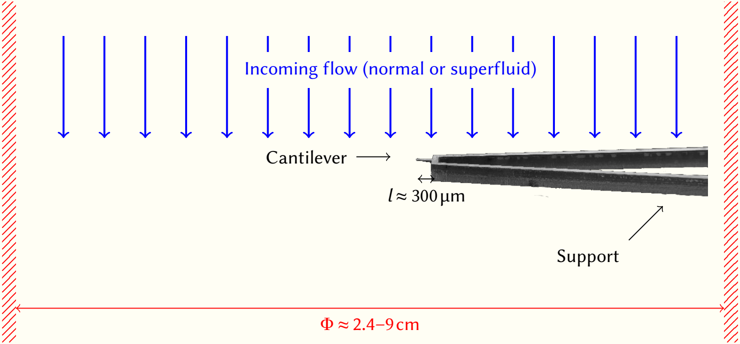
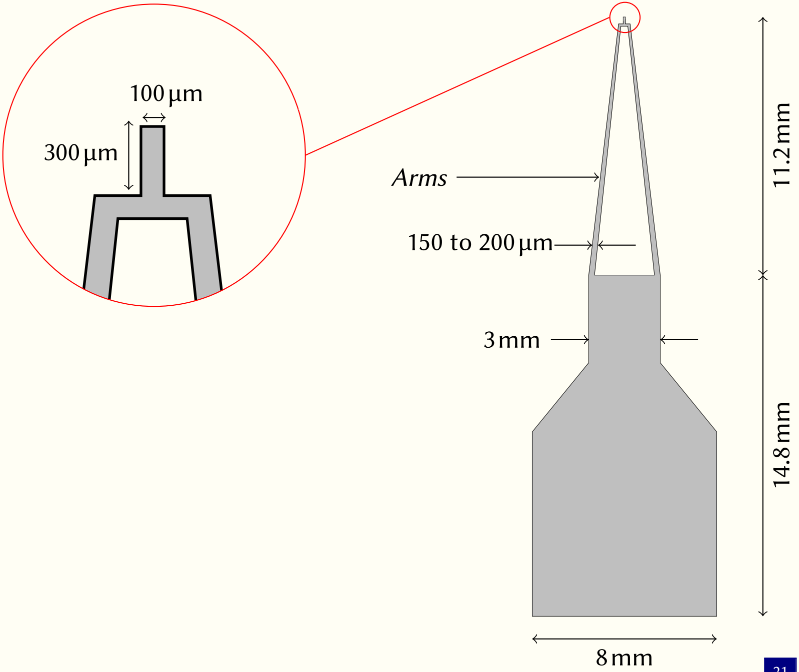
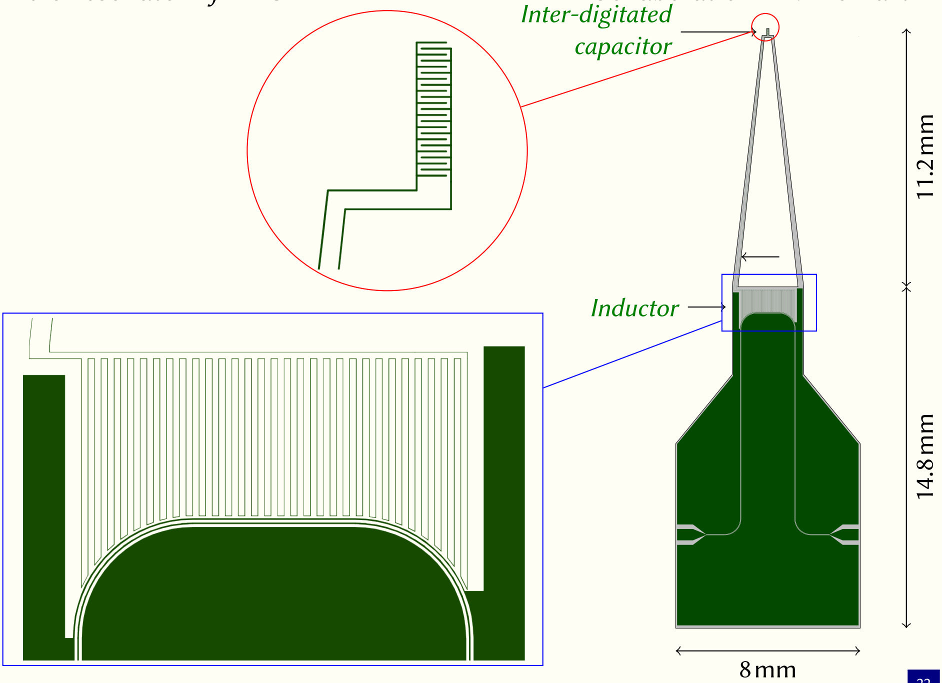
Cantilever anemometry

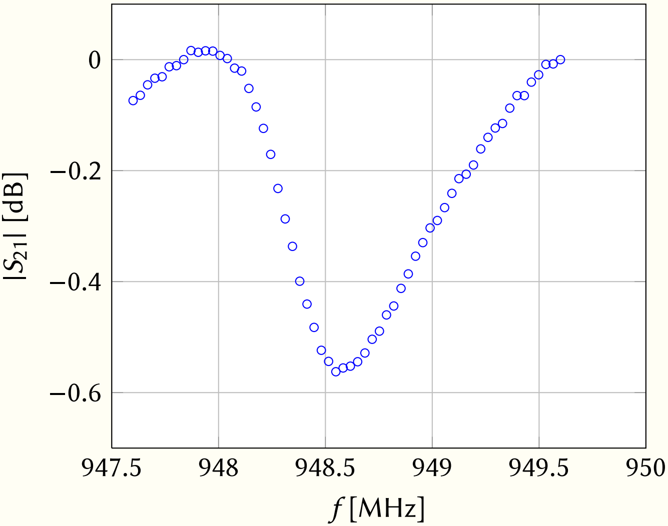


- The cantilever tip is deflected by the incoming flow ;
- Technique was validated in classical turbulence (Barth *et al.*, 2005) ;
- The cantilever has to be inside the bulk of the flow ;
- The arms have to be as transparent as possible.

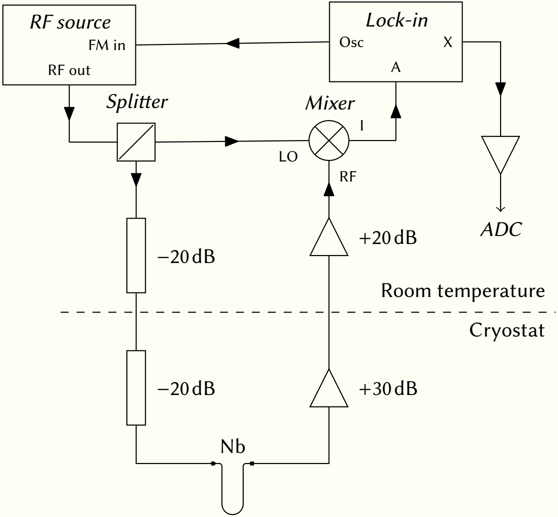

100 µm
300 µm
Arms
150 to 200 µm
3 mm
11.2 mm
14.8 mm
8 mm

Micro-resonator : $f \approx 1\,\mathrm{GHz}$ Collaboration : A. Monfardini

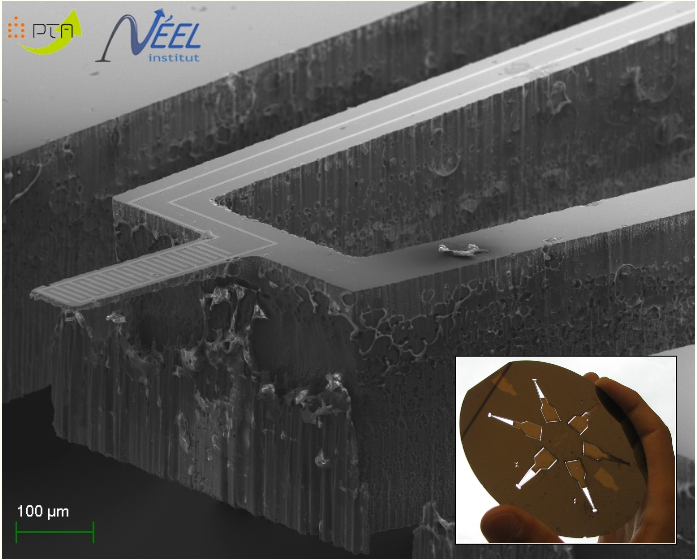




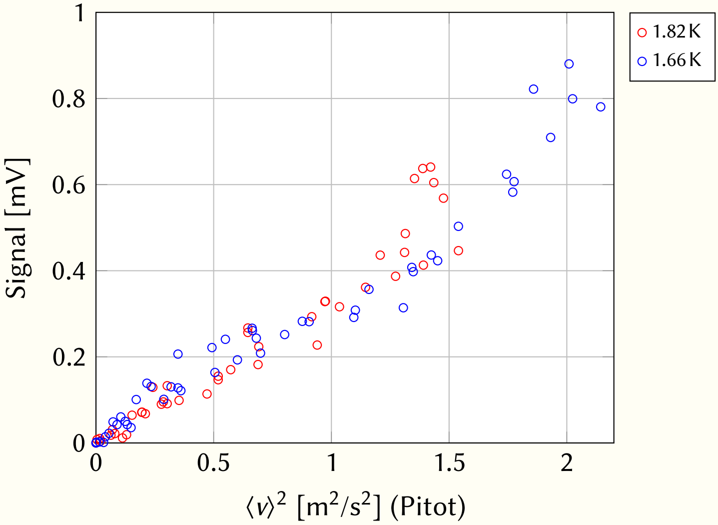
$$\boxed{Q \approx 10^3}$$


RF source
FM in
RF out
Lock-in
Osc
X
A
Splitter
Mixer
I
LO
RF
ADC
−20 dB
+20 dB
Room temperature
Cryostat
−20 dB
+30 dB
Nb

Cantilever on its silicon wafer before splitting

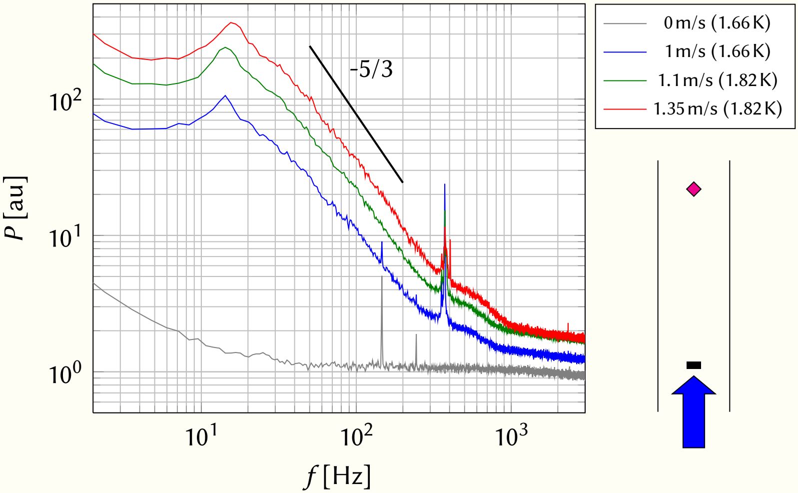


Preliminary calibration of the cantilever probe

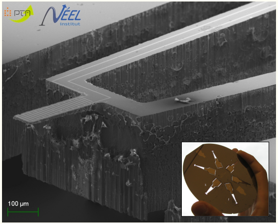


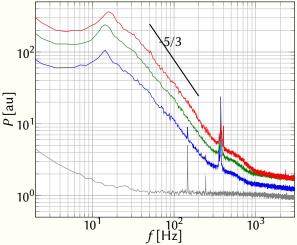
Scaling law fits the expectations

Cantilever : first measurements in He II



The first prototype achieve a spatial resolution comparable to the best micro-Pitots !





Advantages of the cantilever probe

- Less sensitive to acoustic noises
- Easier to miniaturize (fully micro-machined)

Perspectives

- Increase sensitivity
- Improve resolution (at least down to 50 µm)
- Multiplexed cantilever array

1. Introduction

2. Cantilever probe

3. Velocity fluctuations & energy cascade
    - Motivations
    - TSF collaboration
    - Results in Toupie wind tunnel

4. Small-scale behavior & vorticity

5. Conclusion

Velocity fluctuations

Reference measurement (Maurer & Tabeling, 1998)

- -5/3 spectrum in a french washing machine ;
- Similar above and below T_λ ;

Incentive for a new experiment

- Confirm the result and extend it to other geometries ;
- Pressurized wind tunnel to make sure there is no bubble in He I ;
- Carry out a canonical, homogeneous and isotropic flow ;
- Low turbulence intensity to prevent bias on the probe ;
- Direct assessment of the energy cascade : 4/5-law.

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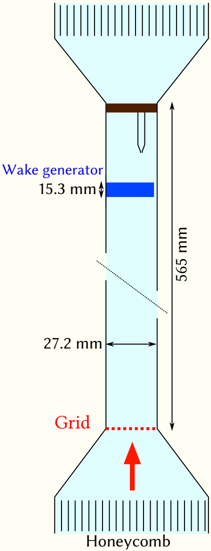
Velocity fluctuations : TSF collaboration

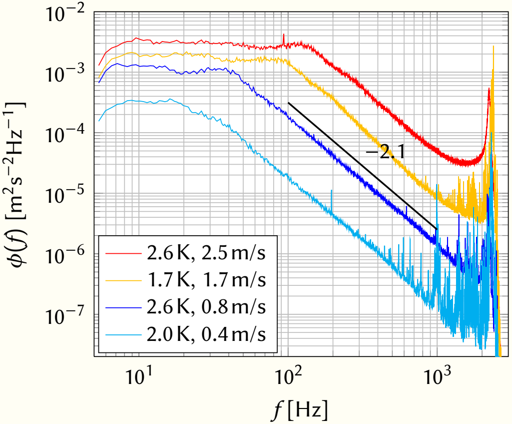
TSF collaboration (ANR & Région Rhône-Alpes financial support)

- SBT/INAC, CEA Grenoble/UJF
- Institut Néel, CNRS/UJF/Grenoble-INP
- LEGI, Grenoble-INP/UJF/CNRS
- SPEC/IRAMIS, CEA Saclay/CNRS
- Laboratoire de Physique, ENSL/CNRS

TSF near-wake flow

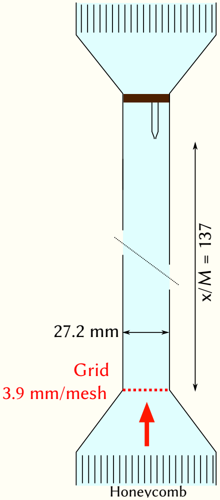


$Re_c = 3 \times 10^5 — 2 \times 10^6$



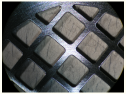
- ✔ Same scaling law in He I and He II ;
- ✔ Pressurized wind tunnel : no bubbles ;
- ✖ Not homogeneous nor isotropic ;
- ✖ High turbulence intensity : bias ;
- ✖ No direct proof of the cascade.

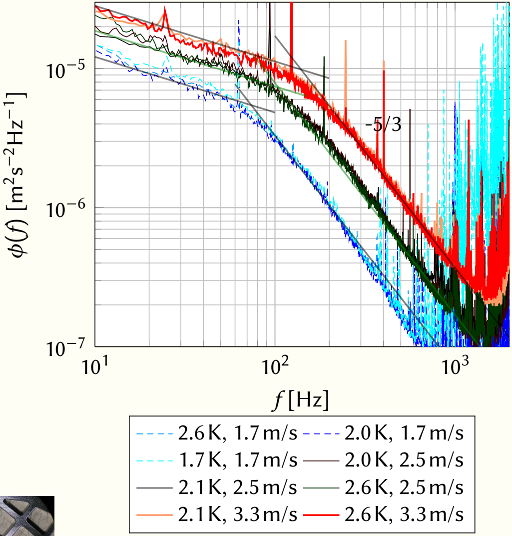
TSF grid flow



$Re_M = 1 \times 10^5 — 2 \times 10^6$

$\tau \approx 1.5\,\%$





Salort *et al*, PoF, 2010

TSF grid flow : Kolmogorov constant

$$\boxed{E(k) = C_k \epsilon^{2/3} k^{-5/3}}$$

Estimate for ϵ

- v_{rms} measurements at two positions ($\Delta x \approx 17M \approx 6\,\mathrm{cm}$)
- Conservation of energy :

$$\epsilon_{\mathrm{production}} = \epsilon_{\mathrm{inertial}} = \epsilon_{\mathrm{dissipation}}$$

- Estimate for ϵ :

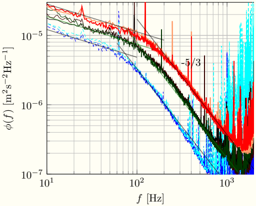
$$\epsilon \approx \left| \frac{\partial v_{\mathrm{rms}}^2}{\partial t} \right| \approx \langle v \rangle^3 \left| \frac{\partial \tau^2}{\partial x} \right| \approx \langle v \rangle^3 \frac{(\tau_2^2 - \tau_1^2)}{(x_2 - x_1)}$$

TSF grid flow : Kolmogorov constant

$$\boxed{E(k) = C_k \epsilon^{2/3} k^{-5/3}}$$

Estimate for C_k

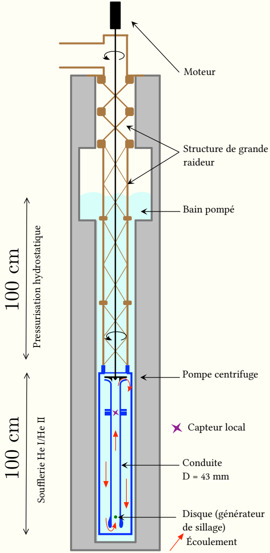
- Classical turbulence literature : $C_k = 1.0$—1.74
- TSF experiment : $C_k = 0.9$—1.2



TSF grid flow

- ✔ Confirmation of the -5/3 spectrum in He I and He II ;
- ✔ Homogeneous and isotropic turbulence in quantitative agreement with the literature (turbulence intensity, integral scale, Kolmogorov constant) ;
- ✖ Low signal-to-noise ratio : higher-order moments (> 2) not well resolved

New wind tunnel : Toupie

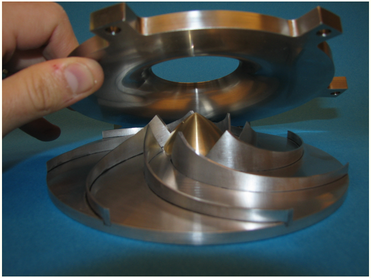


Mechanical design and machining : G. Garde

Specifications

- Flexibility (cold in 2 days) ;
- Rigidity ;
- Static pressurization ;
- Optimized for low temperatures (1.2 K in term) ;
- Low-noise gas heating ;
- Adjustable integral scale.

Optimization example

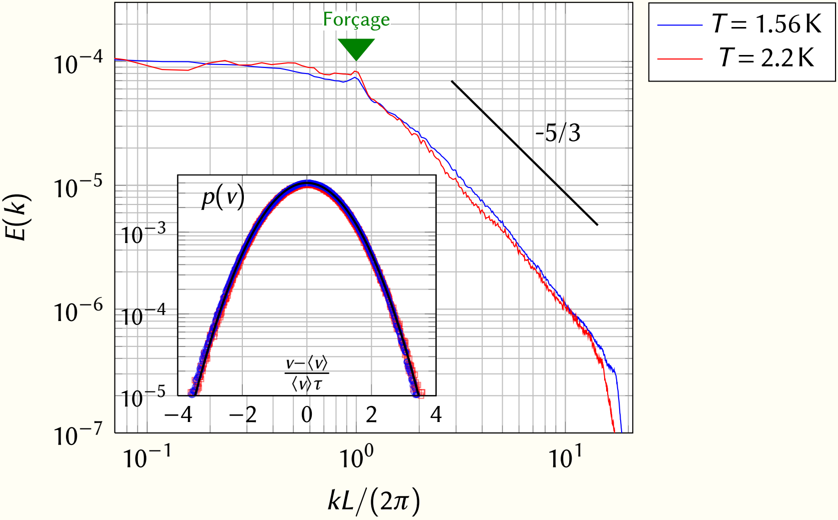


Specifications : ENSE3

Result at first run (primary pump only)

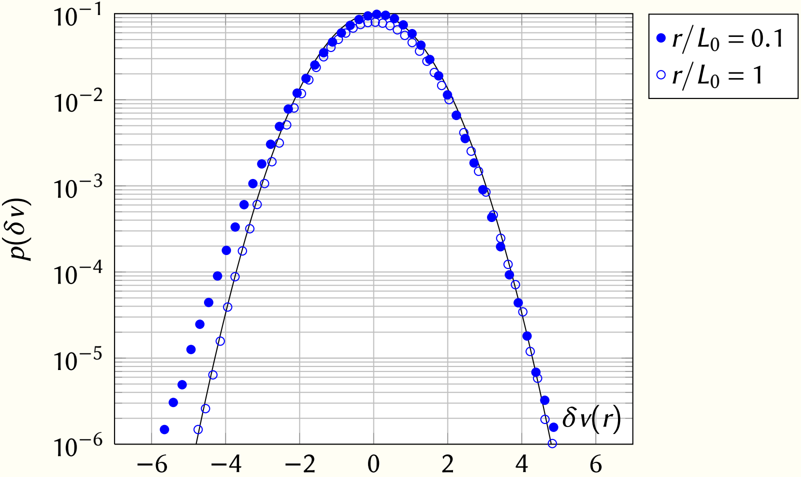
- $\langle v \rangle \approx 1\,\mathrm{m/s}$ at $1.55\,\mathrm{K}$
- Mass-flow rate : $130\,\mathrm{g/s}$ ($\approx 100\,\mathrm{L/min}$ in closed circuit)
  TSF: $700\,\mathrm{g/s}$

Toupie : first results (2011)



$\langle v\rangle = 1.1\,\mathrm{m/s},\ R_\lambda = 1100,\ L/\eta \approx 8.8 \times 10^3$

Turbulent cascade : skewness of the longitudinal velocity increments

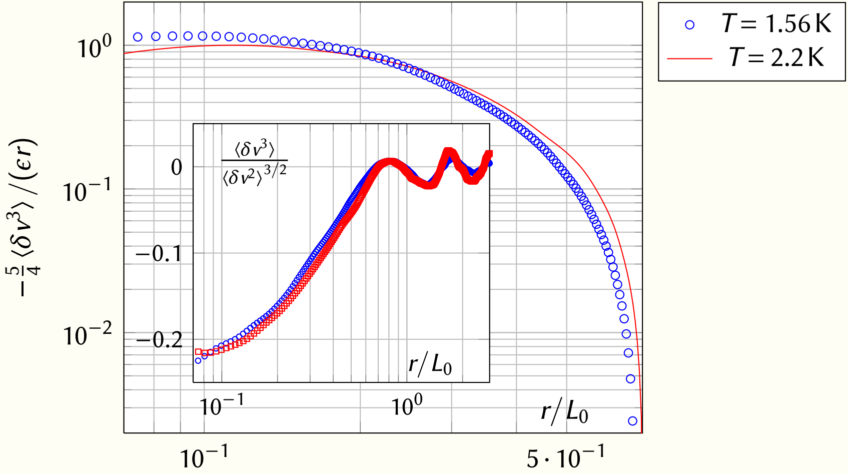


Turbulent cascade : 4/5-law $\langle \delta v^3 \rangle = -\frac{4}{5}\epsilon r$

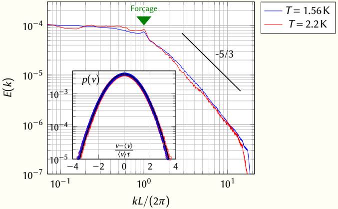
Estimation for ϵ

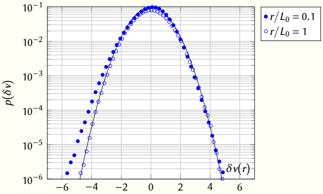
- 4/5-law valid in He I (classical fluid) ;
- Velocity spectrum indistinguishable between He I and He II ;
- Kolmogorov constant identical in He I and He II ;
- Measurements in He I and He II.

Turbulent cascade : 4/5-law $\langle \delta v^3 \rangle = -\frac{4}{5}\epsilon r$



Salort, *et al*, submitted to EPL





Toupie far-wake flow

- ✔ -5/3 spectrum in He I and He II ;
- ✔ Skewness of the longitudinal velocity increments in quantitative agreement with the classical turbulence literature ;
- ✔ 4/5-law : direct assessment of the energy cascade

Similarities Quantum *versus* Classical turbulence

- -5/3 spectrum ;
- Integral quantities : dissipation rate, turbulence intensity, large scale ;
- 4/5-law and higher-order moments

Differences Quantum *versus* Classical turbulence ?

- Equipartition of energy at intermediate scales
- -5/3 vorticity spectrum at 1.55 K

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4. **Small-scale behavior & vorticity**
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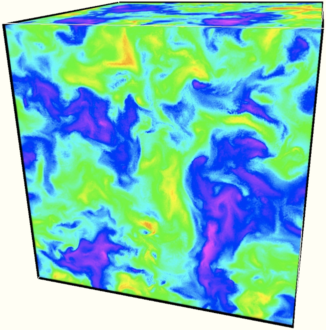
Numerical simulations

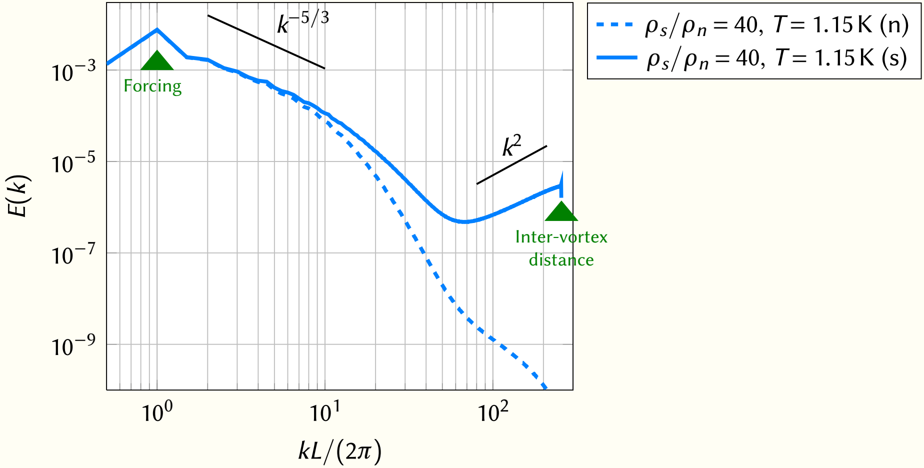
- Continuous two-fluid model :

$$\begin{aligned} \frac{D\vec{v}_n}{Dt} &= -\frac{1}{\rho_n}\nabla p_n + \frac{\rho_s}{\rho}\vec{F}_{ns} + \vec{f}_n^{ext} + \frac{\mu}{\rho_n}\nabla^2\vec{v}_n \\ \frac{D\vec{v}_s}{Dt} &= -\frac{1}{\rho_s}\nabla p_s - \frac{\rho_n}{\rho}\vec{F}_{ns} + \vec{f}_s^{ext} \end{aligned}$$

- Resolved scales :
  Forcing : L — Cut-off : inter-vortex distance δ

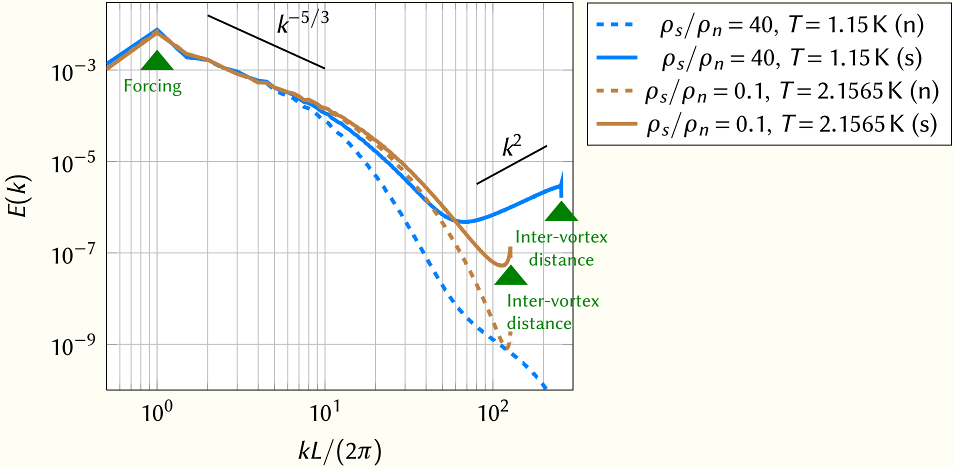


1st difference : spectral behavior (simulations)



$R_\lambda \approx 100$

1^{st} difference : spectral behavior (simulations)



$R_\lambda \approx 100$

2nd difference : enstrophy spectrum ($|\vec{\omega}_s|$)

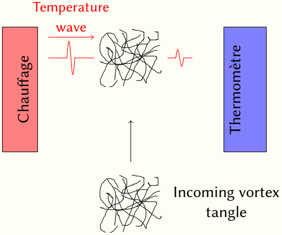
Reference measurement (CRTBT/ESPCI/ESIEE, 2007)

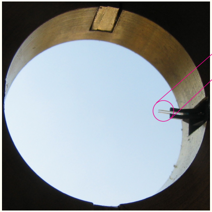
- Second-sound tweezer
  First developments : H. Willaime & P. Tabeling continued in Grenoble
- -5/3 spectrum at 1.55 K
- The interpretation suggests that the slope of the spectrum may change with temperature.

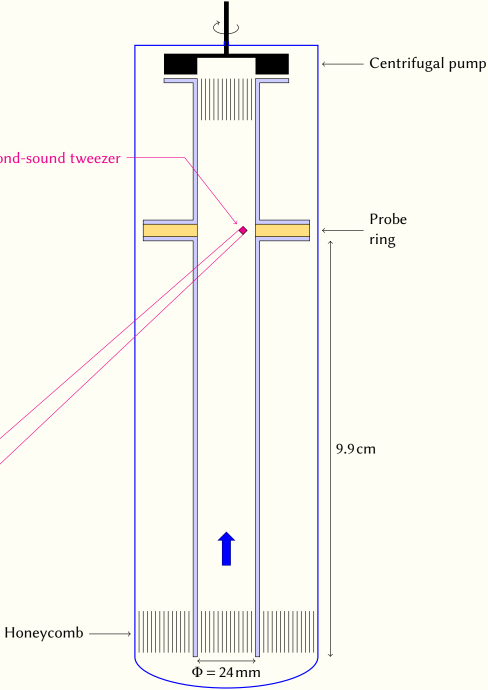
New measurement

- Reproduce and extend to other temperatures ;
- New second-sound tweezer based on a gold-tin thermometer which allows measurements between 1.3 K and 2.2 K (collaboration IEF Orsay).

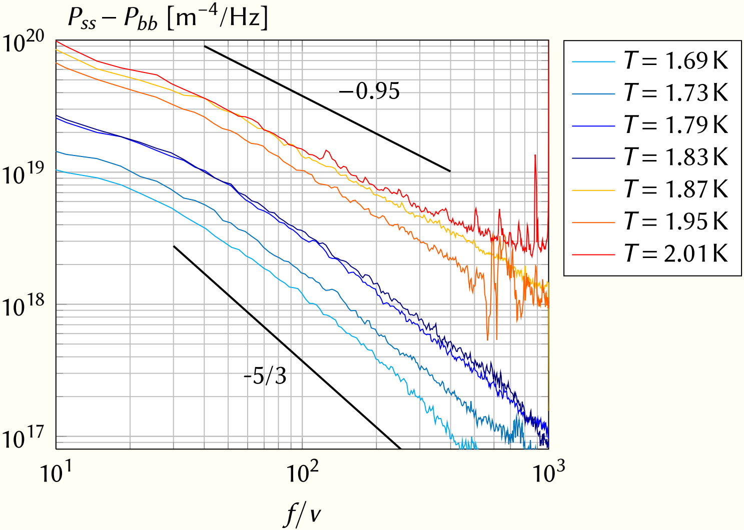
Experimental setup





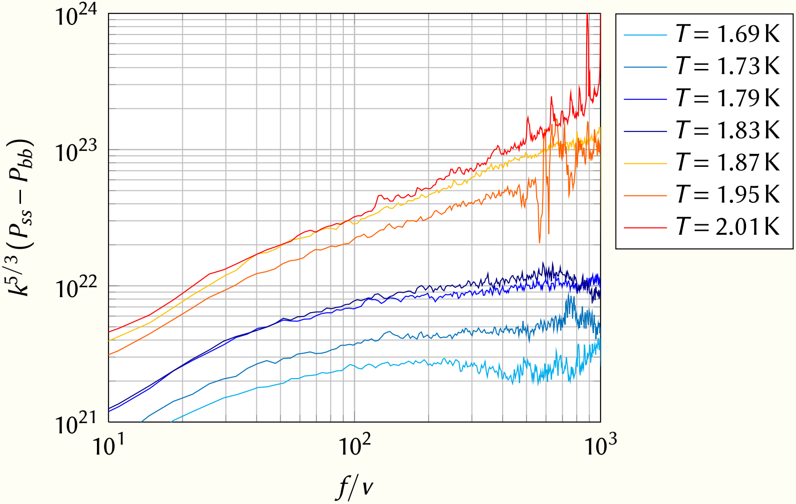


Superfluid vorticity* : temperature dependance

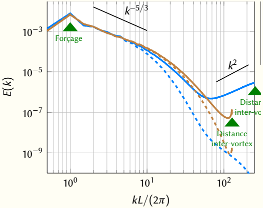


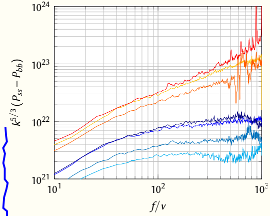
* quantum vortex lines density

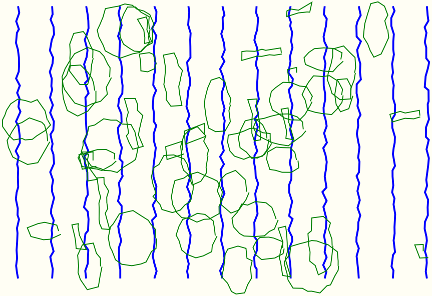
Superfluid vorticity* : temperature dependance



* quantum vortex lines density







Unique interpretation framework

- Locking of the normal/superfluid components at inertial scales
  Polarized field : $\vec{\omega}_\parallel \rightarrow L_\parallel = \left|\vec{\omega}_\parallel\right| / \kappa$
- Small-scale excitations (k^2)
  Random field : $\vec{\omega}_\times \rightarrow L_\times = |\vec{\omega}_\times| / \kappa$

1. Introduction

2. Cantilever probe

3. Velocity fluctuations & energy cascade

4. Small-scale behavior & vorticity

5. Conclusion

Conclusion: Quantum *versus* classical turbulence

Strong analogies at inertial scales

- $k^{-5/3}$ spectra ;
- Kolmogorov constant, turbulence intensity, energy dissipation rate, cascade ;
- Locking of the normal and superfluid components
- (effective viscosity)
- (skewness of the longitudinal velocity increments in the numerical simulations)
- (higher-order moments, intermittency)

New physics at small-scale

- Spectral increase with a k^2 scaling (equipartition)
- Temperature dependance of vorticity spectrum.

Some perspectives

Experimental challenge : measure the k^2 scaling

- Increase the scales : SHREK collaboration
- Decrease the temperature : future of TOUPIE
- Improve the probe resolution : cantilever perspective

Superconducting micro-resonator : a tool for hydrodynamics

- Array of cantilevers (spatial correlations)
- Map of boundary layer density (Rayleigh-Bénard convection)
- Bubble probe (diphasic helium)

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